Math 126 Problem 14: Visual Applications of SVD

Spring 2027

Submission Instructions

Please upload your work to Gradescope by **11:59pm on Monday, March 29, 2027 (Pacific time).**

- You are encouraged to discuss ideas and work with your classmates. However, you *must acknowledge* your collaborators at the top of each solution on which you collaborated with others and you *must write* your solutions independently.
- Your solutions to theory questions must be written legibly, or typeset in LaTeX or markdown.
- I recommend that you write your solutions in a Python notebook using Google Colab.
- You should submit your solutions as a *single PDF* via the assignment on Gradescope.

Grading: The point of the problem set is for *you* to learn. To this end, I hope to disincentivize the use of LLMs by *not* grading your work for correctness. Instead, you will grade your own work by comparing it to my solutions. This self-grade is due the Friday *after* the problem set is due, also on Gradescope.
Original notebook: Google Colab.

Visual Applications of SVD

Part A: Low Rank Approximations to Images

Load an image of your choice from the internet as a matrix $\mathbf{X} \in \mathbb{R}^{n \times d}$. Use `numpy`'s built-in tools to compute its SVD. Plot the singular values by their index; this is known as a *spectrum* plot.

Next, find plot the best rank-k approximation for $k = 1, 5, 10, 50, 100, 500, 1000$. How do the rank-$k$ approximations relate to the spectrum?

Part B: Anomaly Detection

In this problem, we'll consider a different way of building the matrix $\mathbf{X}$. Download a surveillance video (e.g., from here). Each row of $\mathbf{X}$ is a time step where the vector is matrix representation of the corresponding image *flattened*. Note: Please compress the image first so the flattened image is a reasonable size.

Next, compute the SVD of your matrix $\mathbf{X}$. Try subtracting the right singular vector from a row of $\mathbf{X}$ with abnormal movement. What do you see?