Math 126 Problem 1: Law of Large Numbers and Conditional Information

Spring 2027

Submission Instructions

Please upload your work to Gradescope by **11:59pm on Monday, January 25, 2027 (Pacific time).**

- You are encouraged to discuss ideas and work with your classmates. However, you *must acknowledge* your collaborators at the top of each solution on which you collaborated with others and you *must write* your solutions independently.
- Your solutions to theory questions must be written legibly, or typeset in LaTeX or markdown.
- I recommend that you write your solutions in a Python notebook using Google Colab.
- You should submit your solutions as a *single PDF* via the assignment on Gradescope.

Grading: The point of the problem set is for *you* to learn. To this end, I hope to disincentivize the use of LLMs by *not* grading your work for correctness. Instead, you will grade your own work by comparing it to my solutions. This self-grade is due the Friday *after* the problem set is due, also on Gradescope.
Original notebook: Google Colab.

Part A: Law of Large Numbers

Let's consider the global distribution of heights: suppose it's normally distributed (a Gaussian) with mean $\mu = 5.63$ and standard deviation $\sigma = .24$.

Generate $n = 1000$ samples from this distribution and plot a histogram.

Next, consider the distribution of *average* heights in a sample of 100 people. Specifically, sample 100 people, compute their average height, and repeat $n = 1000$ times. Plot the distribution of these average heights on the same histogram as before.

How do the empirical expectations of the two distributions compare? How about their variances?

Part B: Conditional Information

Suppose I tell you that I have two cats. What's the probability that at least one of my cats is a boy when...

- I don't provide any additional information.

- I tell you that one of my cats is a girl (I haven't told you which).
- I tell you that one of my cats is a girl and born on a Sunday (I still haven't told you which).

Why does the probability keep dropping?